Body symmetry – studies in the Framsticks simulator

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Details of this research are available in [\[JK06;](#page-69-0) [JK08\]](#page-69-1).

Model components

[Framsticks](#page-1-0)

[Reminder of the creature](#page-1-0) model

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

Body

- Parts (3D location & orientation)
- Joints
- **o** Brain
	- Neurons (embodied or not)
	- Connections

Model components

[Framsticks](#page-1-0)

[Reminder of the creature](#page-1-0) model

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion

Body

- Parts (3D location & orientation)
- Joints
- **o** Brain
	- Neurons (embodied or not)
	- Connections

Properties

Physical and biological: lengths, sizes, masses, etc.

Model constraints

[Framsticks](#page-1-0)

[Reminder of the creature](#page-1-0) model

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

- at most one Joint can directly connect two Parts
- each Joint must be connected with two distinct Parts
- all Parts must be directly or indirectly connected with each other

Native simulation engine – *MechaStick*

[Framsticks](#page-1-0)

[Reminder of the creature](#page-1-0) model

[Symmetry](#page-6-0)

- physics-based: create real-world feeling to intuitively understand behaviors
- not necessarily very accurate but fast performance matters demo
- Parts: atomic physical objects
- Joints: description of internal forces and constraints, visualized as sticks \bullet

Native simulation engine – *MechaStick*

[Framsticks](#page-1-0)

[Reminder of the creature](#page-1-0) model

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

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Motion

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- **•** physics-based: create real-world feeling to intuitively understand behaviors
- \bullet not necessarily very accurate but fast performance matters $\left($ demo
- Parts: atomic physical objects
- Joints: description of internal forces and constraints, visualized as sticks

—

- rigid bodies: no
- volume bodies: no
- collision detection within creatures: no

Symmetry

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

[References](#page-69-2)

Figure: Vitruvian Man

Symmetry. What's that?

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

Definition

Symmetry is an intrinsic property of a mathematical object which causes it to remain *invariant* under certain classes of transformations (such as rotation, reflection, inversion, or more abstract operations).

Symmetry in various disciplines

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

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Motion [symmetry](#page-47-0)

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[References](#page-69-2)

Figure: The Taj Mahal, Agra, India, 1648 r.

- Physics
- Math
- **•** Music
- Poetry
- **•** Architecture

Symmetry in various disciplines

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

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Motion [symmetry](#page-47-0)

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Figure: The Taj Mahal, Agra, India, 1648 r.

- Physics
- Math
- **•** Music
- Poetry
- **•** Architecture
- Moral symmetry (tit for tat)

Symmetry

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

Herman Weyl, "Symmetry"

Symmetry is an idea which has guided man through the centuries to the understanding and the creation of **order**, **beauty** and **perfection**.

Symmetry in biology

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

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Figure: Symmetry – a popular evolutionary concept.

- Popular evolutionary concept
- Usually bilateral symmetry (the bilateria)
- Oldest known symmetrical organism: Vernanimalcula (600 mln years ago)
- Notable asymmetrical exceptions: sponges.

Sponges

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

Motion

[symmetry](#page-47-0) [Motivations](#page-47-0)

[References](#page-69-2)

Figure: Sponges

Symmetry everywhere?

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[References](#page-69-2)

- Animals are symmetrical only superficially and only in a macro scale
- Asymmetry in chemistry
- Alice's cat
- DNA is clockwise

What is on the other side of looking glass?

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

Figure: On the other side

- Is the reflected world possible?
- Let us reflect the whole universe. . . from stars till atoms. . .

What is on the other side of looking glass?

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[References](#page-69-2)

Figure: On the other side

- **a** Is the reflected world possible?
- **a** Let us reflect the whole universe. . . from stars till atoms. . .
- A reflection of neutrino is $impossible \rightarrow reflected$ world is impossible. . .

What is on the other side of looking glass?

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Motion

Figure: On the other side

- **a** Is the reflected world possible?
- **a** Let us reflect the whole universe from stars till atoms. . .
- A reflection of neutrino is $impossible \rightarrow reflected$ world is impossible. . .
- **Q** unless we also reflect the arrow of time.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

[Motivations](#page-17-0)

Motion [symmetry](#page-47-0)

An open problem.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

Motion [symmetry](#page-47-0)

• An open problem.

Females of some species prefer males with the most symmetrical sexual ornaments.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

Motion

• An open problem.

- Females of some species prefer males with the most symmetrical sexual ornaments.
- For humans, there are proved positive correlations between facial symmetry and health and
- **•** between facial symmetry and perception of beauty

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

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- Intuition: bilateral symmetry resulted from the direction of movement of living creatures

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

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- Proof: positive correlations between locomotive efficiency and morphological symmetry

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

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- Proof: positive correlations between locomotive efficiency and morphological symmetry
- If so, why in the world of flowers symmetry (usually radical) is so common? Certainly not for locomotion.

Numerical measure of symmetry – motivations

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

- Common language is capable to express various degrees of symmetry, but no general numerical symmetry definition exists
- Natural, binary notion of symmetry is insufficient for research
- Numerical measure of symmetry could allow determining the extent to what an object is symmetrical, but also. . .
- if one object is more symmetrical than another.

Numerical measure of symmetry – motivations

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

Motion

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Symmetry is not such a popular concept in artificial worlds, so in order to study the phenomenon of symmetry and its implications, there is a need for defining a **numerical**, fully **automated** and **objective** measure of symmetry for creatures living in artificial environments

More motivations

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Motivations](#page-17-0)

- A tool for researcher (earlier: "similarity" measure)
- Possible research applications:
	- Do symmetrical creatures move faster/further/more reliably?
	- Do symmetrical creatures perform better in environments they were not evolved in?
	- Does evolution produce more symmetrical creatures in worlds with difficult terrain/bigger/smaller gravitation?
	- . . . and more

Creature's model (framsticks)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

[References](#page-69-2)

Only skeleton is taken into account.

Solid 3D objects

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Static [symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

[References](#page-69-2)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Static [symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

• The Symmetry Condition. If c is perfectly bilaterally symmetrical, then $sym(c) = 1.0$.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

- **The Symmetry Condition.** If c is perfectly bilaterally symmetrical, then $sym(c) = 1.0$.
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[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Approach](#page-26-0)

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- \bullet **The Common Sense Condition.** If c_1 is more symmetrical than c_2 , then $sym(c_1) > sym(c_2)$.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Approach](#page-26-0)

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- **The Proportional Difference Condition.** The difference between $sym(c_1)$ and $sym(c_2)$ should correspond to the difference in anatomical symmetry between c_1 and c_2 .

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[Approach](#page-26-0)

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- **The Proportional Difference Condition.** The difference between $sym(c_1)$ and $sym(c_2)$ should correspond to the difference in anatomical symmetry between c_1 and c_2 .
- **The Scalability Condition.** The proposed measure should be robust against scaling: for creature c_2 that is a scaled version of c_1 (body enlarged or diminished), we expect $sym(c_2) = sym(c_1)$.

[Symmetry](#page-6-0)

[Approach](#page-26-0)

Motion

Let us denote symmetry of a creature c about plane p as $sym(c, p)$. We say that "a creature is symmetrical" if it is symmetrical about **any plane**, therefore we are looking for a plane that yields the highest symmetry:

$$
sym(c) = \max_{p} (sym(c, p))
$$
 (1)

Creature's model

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

Looking for matching sticks. . .

How to compute $sym(c, p)$? (1)

*s*4

*s*5

How to compute $sym(c, p)$? (2)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

How to compute $sym(c, p)$? (3)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Approach](#page-26-0)

Motion [symmetry](#page-47-0)

$$
sym(c, p) = \max_{\Pi} \left(\frac{\sum_{(s_1, s_2) \in \Pi} w_{s_1 s_2} sim(s_1, s_2)}{\sum_{(s_1, s_2) \in \Pi} w_{s_1 s_2}} \right)
$$

where

$$
\mathsf{w}_{\mathsf{s}_1\mathsf{s}_2} = \left\{ \begin{array}{ccc} \mathsf{len}(\mathsf{s}_1) + \mathsf{len}(\mathsf{s}_2) & \text{if} & \mathsf{s}_1 \neq \mathsf{s}_2 \\ \mathsf{len}(\mathsf{s}_1) & \text{if} & \mathsf{s}_1 = \mathsf{s}_2 \end{array} \right.
$$

$$
sim(s_1, s_2) = exp \frac{-dist^2(s_1, s_2)}{(\alpha \cdot s_f)^2}
$$
 (4)

(2)

(3)

where α is a constant, and s_f is a creature scale factor.

Sample landscape

[Symmetry](#page-6-0)

[Approach](#page-26-0)

Motion

Figure: In order to find the plane of the highest symmetry, we sample the 3-dimensional (α, β, t) space for each creature stick and then perform a local search to further improve the best found plane.

Illustration of symmetry planes (1)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Experiments](#page-39-0)

Motion

Figure: Sample creatures, estimation of their symmetry planes and symmetry values. Values of symmetry are: 1.0, 1.0, 0.99

Illustration of symmetry planes (2)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

[Experiments](#page-39-0)

Motion [symmetry](#page-47-0)

[References](#page-69-2)

Figure: Values of symmetry are: 0.97, 0.82

Illustration of symmetry planes (3)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

[Experiments](#page-39-0)

Motion [symmetry](#page-47-0)

Figure: Values of symmetry are: 0.70, 0.39

Illustration of symmetry planes (4)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Static [symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

[References](#page-69-2)

A random set of individuals

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

[Experiments](#page-39-0)

Motion

Figure: 30 diverse creatures arranged horizontally according to their values of symmetry (the most symmetrical on the right).

Symmetry in human design and evolution

Figure: Distribution of symmetry values among 84 creatures (38 designed, 46 evolved).

Evolved creatures

[Symmetry](#page-6-0) itself

Motion

Figure: Evolved creatures. Constructs with the highest symmetry are usually simple.

Designed creatures

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Motivations](#page-47-0)

Figure: Designed creatures with symmetry of 1.0.

Motivations

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

-
-
-

[Motivations](#page-47-0)

-
-

- Operates on the phenotype in motion (opposed to: symmetry of genotype)
- Characterizes motion (a feature of the motion pattern).
- Other: whether (to what degree) the movement is periodic or chaotic, how dynamic, effective it is
- Implications:
	- understanding the evolution on Earth
	- methods of locomotion both in living animals and designed robots

Static symmetries

Figure: Symmetry planes of the four considered creatures. Symmetry values are given in brackets.

3D paths

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

Static [symmetry](#page-17-0)

-
-
-

Motion

[Approach](#page-48-0)

Figure: Sample 3D paths for four creatures.

2D paths

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

itself

[symmetry](#page-17-0)

Motion

[Approach](#page-48-0)

Figure: 10 paths for four considered creatures.

Symmetry (3df) over time

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Approach](#page-48-0)

Figure: The values of symmetry over time.

2D paths with symmetries

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Approach](#page-48-0)

Figure: The creature 2D paths (red) with vertical planes shown (green).

Smoothed paths

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Figure: The original paths (red) and the ones smoothed using a low pass filter (blue).

Movement directions

Figure: Movement directions based on the smoothed paths over time.

Vertical (1df) symmetry over time

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

[Approach](#page-48-0)

Figure: The values of vertical (1df) symmetry over time.

Static symmetries

Figure: Symmetry planes of the four considered creatures. Symmetry values are given in brackets.

Final symmetry values (soft 1df symmetry)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Static [symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Approach](#page-48-0)

Table: Soft dynamic 1df symmetries (soft 1df), their standard deviations and maximal and minimal values.

[Framsticks](#page-1-0)

[Symmetry](#page-6-0) itself

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Approach](#page-48-0)

• MechaStick

experiments in 2001: diverse ways of movement evolved

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion

[symmetry](#page-47-0)

[Approach](#page-48-0)

- experiments in 2001: diverse ways of movement evolved
- were they really diverse?
- mostly simple creatures (a few sticks. . . large constructs are inefficient)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Approach](#page-48-0)

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- most interesting ones were designed by hand and NNs were evolved

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[Approach](#page-48-0)

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[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion

[Approach](#page-48-0)

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ODE

high expectations (accuracy, volume bodies, self-collisions)

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Approach](#page-48-0)

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	- high expectations (accuracy, volume bodies, self-collisions)
	- evolving movement turned out to be even more difficult! :o
	- elasticity of MechaStick was **so** important!

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Approach](#page-48-0)

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[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Approach](#page-48-0)

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[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Approach](#page-48-0)

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[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Approach](#page-48-0)

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	- sticks as cuboids: instability of simulation, oscillations, and... rolling ("active")
	- many simulation parameters, each of them is important
	- interdependence between mass, gravity, collision parameters, muscle strength and speed
	- rolling is a local optimum (so far) $\left($ demo
- **o** lots of lessons learned... and weeks of simulation.

Further research

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

Motion

[Further research](#page-68-0)

- For which objectives (speed and locomotion, predation, height, etc.) evolution promotes symmetrical creatures?
- Is symmetry beneficial for creatures evolved spontaneously?
- \bullet Does symmetry emerge for creatures evolved spontaneously? (evolve, observe, surprise!)
- Which genetical encodings promote symmetry?
- Symmetry as a component of fitness formula.
- Encoding that preserves symmetry. Comparison with other encodings.

References I

[Framsticks](#page-1-0)

[Symmetry](#page-6-0)

[symmetry](#page-17-0)

Motion [symmetry](#page-47-0)

[References](#page-69-2)

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